Resol els sistemes calculant la matriu inversa de la matriu de coeficients

1.- $\begin{cases} x+y=5 \\ x-2y=2 \end{cases}$

Escrivim el sistema en forma de matriu

$$\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Calculem la matriu inversa de la matriu de coeficients

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & -2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} [1] \\ 1[2]-1[1] \end{array}$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & -3 & -1 & 1 \end{array}\right] \quad \begin{array}{l} -3[1]-1[2] \\ [2] \end{array}$$

$$\left[\begin{array}{cccc} -3 & 0 & -2 & -1 \\ 0 & -3 & -1 & 1 \end{array}\right] \quad \begin{array}{l} \text{Dividim per -3} \\ \text{Dividim per -3} \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 1/3 & -1/3 \end{array}\right] \quad \text{2 últimes columnes}$$

$$\left[\begin{array}{cc} 2/3 & 1/3 \\ 1/3 & -1/3 \end{array}\right]$$

Multipliquem per l'esquerra per la matriu inversa calculada

$$\frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

L'esquerra de l'igual és la matriu de les incògnites, fem les operacions de la dreta

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Les solucions dels sistema són x=4 i y=1

2.- $\begin{cases} x=5+y \\ 3y+1=x \end{cases}$

Primer ordenem el sistema i l'escrivim en forma de matriu

$$\begin{cases} x=5+y \\ 3y+1=x \end{cases} \Rightarrow \begin{cases} x-y=5 \\ -x+3y=-1 \end{cases}$$

en forma de matriu

$$\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

calculem la matriu inversa de la matriu de coeficients

$$\left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right] \qquad \begin{array}{l} [1] \\ 1[2]+1[1] \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \qquad \begin{array}{l} 2[1]+1[2] \\ [2] \end{array}$$

$$\left[\begin{array}{cccc} 2 & 0 & 3 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right] \qquad \begin{array}{l} \text{Dividim per 2} \\ \text{Dividim per 2} \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 3/2 & 1/2 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right] \qquad \text{2 últimes columnes}$$

$$\left[\begin{array}{cc} 3/2 & 1/2 \\ 1/2 & 1/2 \end{array}\right]$$

Multipliquem per la matriu inversa

$$\frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

obtenim com resultats

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 14 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

3.- $\begin{cases} x + y = 3 \\ y + z = 5 \\ x + z = 4 \end{cases}$

En forma de matriu

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$$

Càlcul de la matriu inversa

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} [1] \\ [2] \\ 1[3]-1[1] \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{bmatrix} \quad \begin{matrix} [1] \\ [2] \\ 1[3]+1[2] \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{bmatrix} \quad \begin{matrix} [1] \\ 2[2]-1[3] \\ [3] \end{matrix}$$

$$\begin{bmatrix} 2 & 2 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{bmatrix} \quad \begin{matrix} 2[1]-2[2] \\ [2] \\ [3] \end{matrix}$$

$$\begin{bmatrix} 4 & 0 & 0 & 2 & -2 & 2 \\ 0 & 2 & 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{bmatrix} \quad \begin{matrix} \text{Dividim per 4} \\ \text{Dividim per 2} \\ \text{Dividim per 2} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{bmatrix} \quad \text{3 últimes columnes}$$

$$\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}$$

Multipliquem per l'esquerra per la matriu inversa

$$\frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$$

i el resultat del sistema és

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

4.- $\begin{cases} x + y + z = 0 \\ 2x + y + z = 2 \\ y + 2z = -2 \end{cases}$

Hem de calcular la matriu inversa de la matriu de coeficients

$$\left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} [1] \\ 1[2]\ -2[1] \\ [3] \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} [1] \\ [2] \\ -[3]\ -1[2] \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \end{array}\right] \quad \begin{array}{l} -[1]\ -1[3] \\ -[2]\ +1[3] \\ [3] \end{array}$$

$$\left[\begin{array}{cccccc} -1 & -1 & 0 & -3 & 1 & 1 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & -1 & 2 & -1 & -1 \end{array}\right] \quad \begin{array}{l} 1[1]\ +1[2] \\ [2] \\ [3] \end{array}$$

$$\left[\begin{array}{cccccc} -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & -1 & 2 & -1 & -1 \end{array}\right] \quad \begin{array}{l} \text{Dividim per -1} \\ \text{Dividim per 1} \\ \text{Dividim per -1} \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right] \quad \text{3 últimes columnes}$$

$$\left[\begin{array}{ccc} -1 & 1 & 0 \\ 4 & -2 & -1 \\ -2 & 1 & 1 \end{array}\right]$$

El producte per la matriu inversa dóna

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 \\ 4 & -2 & -1 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

Els quatre sistemes tenen les mateixes matrius de coeficients que els sistemes anteriors. No cal tornar a calcular la matriu inversa, només hem de canviar la matriu de termes independents

1.- $\begin{cases} x + y = 9 \\ x - 2y = 5 \end{cases}$

La solució és

$$\frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 9 \\ 5 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 23 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{23}{3} \\ \frac{4}{3} \end{pmatrix}$$

2.- $\begin{cases} x = y + 7 \\ 3y + 3 = x \end{cases}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 7 \\ -3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 18 \\ 4 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$$

3.- $\begin{cases} x+y=1 \\ y+z=2 \\ x+z=1 \end{cases}$

$$\frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

4.- $\begin{cases} x+y+z=4 \\ 2x+y+z=5 \\ y+2z=1 \end{cases}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 \\ 4 & -2 & -1 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$$