**Càlcul integral**

1. $\int \frac{dx}{(x-1)^2}$

$$\int \frac{dx}{(x-1)^2} = \begin{bmatrix} x-1=t \\ dx=dt \end{bmatrix} = \int t^{-2}dt = \frac{1}{-1}t^{-1} = -\frac{1}{(x-1)}$$

2. $\int \frac{5x}{\sqrt{1+x^2}}dx$

$$\int \frac{5x}{\sqrt{1+x^2}}dx = \begin{bmatrix} 1+x^2=t \\ 2xdx=dt \end{bmatrix} = \frac{5}{2}\int \frac{dt}{\sqrt{t}} = \frac{5}{2}\int t^{-\frac{1}{2}}dt = \frac{5}{2}\frac{1}{\frac{1}{2}}t^{\frac{1}{2}} = 5\sqrt{t} = 5\sqrt{1+x^2}$$

3. $\int x\sqrt{1+x^2}dx$

$$\int x\sqrt{1+x^2}dx = \begin{bmatrix} 1+x^2=t \\ 2xdx=dt \end{bmatrix} = \int \frac{\sqrt{t}dt}{2} = \frac{1}{2}\frac{1}{\frac{3}{2}}t^{\frac{3}{2}} = \frac{\sqrt{t^3}}{3} = \frac{\sqrt{\left(1+x^2\right)^3}}{3}$$

4. $\int \frac{\ln x}{x}dx$

$$\int \frac{\ln x}{x}dx = \begin{bmatrix} \ln x=t \\ \frac{1}{x}dx=dt \end{bmatrix} = \int tdt = \frac{t^2}{2} = \frac{\ln^2 x}{2}$$

5. $\int \frac{\ln^2 x}{x}dx$

$$\int \frac{\ln^2 x}{x}dx = \begin{bmatrix} \ln x=t \\ \frac{1}{x}dx=dt \end{bmatrix} = \int t^2dt = \frac{t^3}{3} = \frac{\ln^3 x}{3}$$

6. $\int \frac{1}{x\ln^2 x}dx$

$$\int \frac{1}{x\ln^2 x}dx = \begin{bmatrix} \ln x=t \\ \frac{1}{x}dx=dt \end{bmatrix} = \int \frac{1}{t^2}dt = \frac{1}{-1}t^{-1} = -\frac{1}{t} = -\frac{1}{\ln x}$$

7. $\int \sqrt[3]{3x+5}dx$

$$\int \sqrt[3]{3x+5}dx = \begin{bmatrix} 3x+5=t \\ 3dx=dt \end{bmatrix} = \frac{1}{3}\int t^{\frac{1}{3}}dt = \frac{1}{3}\frac{1}{\frac{4}{3}}t^{\frac{4}{3}} = \frac{1}{4}\sqrt[3]{t^4} = \frac{\sqrt[3]{(3x+5)^4}}{4}$$

8. $\int \frac{2x}{x^2+5}dx$

$$\int \frac{2x}{x^2+5}dx = \begin{bmatrix} x^2+5=t \\ 2xdx=dt \end{bmatrix} = \int \frac{1}{t}dt = \ln t = \ln(x^2+5)$$

9. $\int \frac{1}{ax+b}dx$

$$\int \frac{1}{ax+b}dx = \begin{bmatrix} ax+b=t \\ adx=dt \end{bmatrix} = \frac{1}{a}\int \frac{1}{t}dt = \frac{1}{a}\ln t = \frac{\ln(ax+b)}{a}$$

10. $\int \frac{x}{x^2+9}dx$

$$\int \frac{x}{x^2+9}dx = \begin{bmatrix} x^2+9=t \\ 2xdx=dt \end{bmatrix} = \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\ln t = \frac{1}{2}\ln(x^2+9)$$

11. $\int \frac{1}{x\ln x}dx$

$$\int \frac{1}{x\ln x}dx = \begin{bmatrix} \ln x=t \\ \frac{1}{x}dx=dt \end{bmatrix} = \int \frac{1}{t}dt = \ln t = \ln(\ln x)$$

12. $\int \frac{\tan\sqrt{x}}{\sqrt{x}}dx$

$$\int \frac{\tan\sqrt{x}}{\sqrt{x}}dx = \begin{bmatrix} \sqrt{x}=t \\ \frac{1}{2\sqrt{x}}dx=dt \end{bmatrix} = 2\int \tan t dt = 2\int \frac{\sin t}{\cos t}dt = \begin{bmatrix} \cos t=u \\ -\sin t dt=du \end{bmatrix} =$$

$$= 2\int \frac{-du}{u} = -2\ln u = -2\ln(\cos t) = -2\ln(\cos\sqrt{x})$$

13. $\int xe^{-x^2}dx$

$$\int xe^{-x^2}dx = \begin{bmatrix} -x^2=t \\ -2xdx=dt \end{bmatrix} = -\frac{1}{2}\int e^t dt = -\frac{1}{2}e^t = -\frac{1}{2}e^{-x^2}$$

14. $\int \frac{1}{x^2}e^{\frac{1}{x}}dx$

$$\int \frac{1}{x^2}e^{\frac{1}{x}}dx = \begin{bmatrix} \frac{1}{x}=t \\ -\frac{1}{x^2}dx=dt \end{bmatrix} = -\int e^t dt = -e^t = -e^{\frac{1}{x}}$$

15. $\int 5x\cos(x^2+3)dx$

$$\int 5x\cos(x^2+3)dx = \begin{bmatrix} x^2+3=t \\ 2xdx=dt \end{bmatrix} = \frac{5}{2}\int \cos t dt = \frac{5}{2}\sin t = \frac{5}{2}\sin(x^2+3)$$

16. $\int \frac{dx}{\sqrt{x}\cos^2\sqrt{x}}$

$$\int \frac{dx}{\sqrt{x}\cos^2\sqrt{x}} = \begin{bmatrix} \sqrt{x}=t \\ \frac{1}{2\sqrt{x}}dx=dt \end{bmatrix} = 2\int \frac{dt}{\cos^2 t} = 2\tan t = 2\tan\sqrt{x}$$

17. $\int \frac{x}{\sqrt[3]{1+x^2}}dx$

$$\int \frac{x}{\sqrt[3]{1+x^2}}dx = \begin{bmatrix} 1+x^2=t \\ 2xdx=dt \end{bmatrix} = \frac{1}{2}\int \frac{dt}{\sqrt[3]{t}} = \frac{1}{2}\int t^{-\frac{1}{3}}dt = \frac{1}{2}\frac{1}{\frac{2}{3}}t^{\frac{2}{3}} = \frac{3\sqrt[3]{t^2}}{4} = \frac{3\sqrt[3]{(1+x^2)^3}}{4}$$

18. $\int \frac{7x}{x^2+1}dx$

$$\int \frac{7x}{x^2+1}dx = \begin{bmatrix} x^2+1=t \\ 2xdx=dt \end{bmatrix} = 7\int \frac{1}{2}\frac{1}{t}dt = \frac{7}{2}\ln t = \frac{7}{2}\ln(x^2+1)$$

19. $\int 2\sqrt{x^2+x^4}dx$

$$\int 2\sqrt{x^2+x^4}dx = 2\int \sqrt{x^2\left(1+x^2\right)}dx = 2\int x\sqrt{1+x^2}dx = \begin{bmatrix} 1+x^2=t \\ 2xdx=dt \end{bmatrix} =$$

$$= 2\frac{1}{2}\int \sqrt{t}dt = \frac{1}{\frac{3}{2}}t^{\frac{3}{2}} = \frac{2\sqrt{\left(1+x^2\right)^3}}{3}$$

20. $\int \frac{dx}{(6x+5)^3}$

$$\int \frac{dx}{(6x+5)^3} = \begin{bmatrix} 6x+5=t \\ 6dx=dt \end{bmatrix} = \frac{1}{6}\int t^{-3}dt = \frac{1}{6}\frac{1}{-2}t^{-2} = -\frac{1}{12t^2} = -\frac{1}{12(6x+5)^2}$$