

Derivadas

1. Calcula las derivadas

$$y = 4x^3$$

$$y = 2x - x^4$$

$$y = 2x - 6 + 3x^2$$

$$y = \frac{x^3}{4}$$

$$y = \frac{x^2}{2} + \frac{3x^5}{4}$$

$$y = \frac{1}{x}$$

$$y = \frac{4}{x^2}$$

$$y = \sqrt{x}$$

$$y = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$$

$$y = \frac{1}{\sqrt{x}}$$

Handwritten solutions for the first 10 problems on grid paper:

1- $y = 4x^3$	$y' = 12x^2$
2- $y = 2x - x^4$	$y' = 2 - 4x^3$
3- $y = 2x - 6 + 3x^2$	$y' = 2 + 6x$
4- $y = \frac{x^3}{4} = \frac{1}{4}x^3$	$y' = \frac{3}{4}x^2$
5- $y = \frac{x^2}{2} + \frac{3x^5}{4} = \frac{1}{2}x + \frac{3}{4}x^5$	$y' = \frac{1}{2} + \frac{15x^4}{4}$
6- $y = \frac{1}{x} = x^{-1}$	$y' = -x^{-2} = -\frac{1}{x^2}$
7- $y = \frac{4}{x^2} = 4x^{-2}$	$y' = -8x^{-3} = -\frac{8}{x^3}$
8- $y = \sqrt{x} = x^{1/2}$	$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$
9- $y = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} = x^{-1} + x^{-2} + x^{-3}$	$y' = -x^{-2} - 2x^{-3} - 3x^{-4}$ $= -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4}$
10- $y = \frac{1}{\sqrt{x}} = x^{-1/2}$	$y' = -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

Calcula las derivadas

$$y = \sqrt[3]{x}$$

$$y = \frac{3}{\sqrt[3]{x}}$$

$$y = x^3 + 4x^2 - \sqrt{x}$$

$$y = \frac{7\sqrt{x}}{3}$$

$$y = \frac{1}{6\sqrt{x}}$$

$$y = x^{4,6}$$

$$y = 4x^5 + 3x - \frac{4}{x}$$

$$y = x + 23 - x$$

$$y = \frac{x^3}{\sqrt{x}}$$

$$y = x \cdot x^4$$

11.- $y = \sqrt[3]{x} = x^{1/3}$	$y' = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$
12.- $y = \sqrt[3]{\frac{3}{x}} = 3x^{-1/3}$	$y' = 3(-\frac{1}{3}) x^{-4/3} = -\frac{1}{\sqrt[3]{x^4}}$
13.- $y = x^3 + 4x^2 - \sqrt{x}$	$y' = 3x^2 + 8x - \frac{1}{2\sqrt{x}}$
14.- $y = \frac{7\sqrt{x}}{3} = \frac{7}{3} x^{1/2}$	$y' = \frac{7}{3} \cdot \frac{1}{2} x^{-1/2} = \frac{7}{6\sqrt{x}}$
15.- $y = \frac{1}{6\sqrt{x}} = \frac{1}{6} x^{-1/2}$	$y' = \frac{1}{6} (-\frac{1}{2}) x^{-3/2} = -\frac{1}{12\sqrt{x^3}}$
16.- $y = x^{4,6}$	$y' = 4,6 x^{3,6}$
17.- $y = 4x^5 + 3x - \frac{4}{x^2}$	$y' = 20x^4 + 3 + 4x^{-3}$ $= 20x^4 + 3 + \frac{4}{x^3}$
18.- $y = x + 23 - x$	$y' = 0$
19.- $y = \frac{x^3}{\sqrt{x}} = x^{3-1/2} = x^{5/2}$	$y' = \frac{5}{2} x^{3/2} = \frac{5}{2\sqrt{x^3}}$
20.- $y = x \cdot x^4 = x^5$	$y' = 5x^4$

2. Calcula les derivades dels productes i quocients

$$y = x^3(2x+1)$$

$$y = \frac{x^2 + x}{x+1}$$

$$y = (2x+1)(3x-3)$$

$$y = \frac{x^3}{x-1}$$

$$y = (x^2 + 1)(2x+5)$$

$$y = \frac{x+x^3}{x-1}$$

$$y = \frac{x+1}{x}$$

$$y = \frac{x}{x^2 + 1}$$

$$y = \frac{2x+1}{x}$$

$$y = \frac{2x+1}{\sqrt{x}}$$

1.-	$y = x^3(2x+1)$	$y' = 3x^2(2x+1) + 2x^3$
2.-	$y = (2x+1)(3x-3)$	$y' = 2(3x-3) + 3(2x+1)$
3.-	$y = (x^2+1)(2x+5)$	$y' = 2x(2x+5) + 2(x^2+1)$
4.-	$y = \frac{x+1}{x}$	$y' = \frac{x-(x+1)}{x^2} = \frac{-1}{x^2}$
5.-	$y = \frac{2x+1}{x}$	$y' = \frac{2x-(2x+1)}{x^2} = \frac{-1}{x^2}$
6.-	$y = \frac{x^2+x}{x+1}$	$y' = \frac{(2x+1)(x+1)-(x^2+x)}{(x+1)^2}$
7.-	$y = \frac{x^3}{x-1}$	$y' = \frac{3x^2(x-1)-x^3}{(x-1)^2}$
8.-	$y = \frac{x+x^3}{x-1}$	$y' = \frac{(1+3x^2)(x-1)-(x+x^3)}{(x-1)^2}$
9.-	$y = \frac{x}{x^2+1}$	$y' = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$
10.-	$y = \frac{2x+1}{\sqrt{x}}$	$y' = \frac{2\sqrt{x} - (2x+1)\frac{1}{2\sqrt{x}}}{x}$

3. Recorda la derivada de $\sin x$ i aplica la regla de la cadena

$$y = \sin 2x$$

$$y = \sin(3x^5)$$

$$y = \sin \frac{x}{5}$$

$$y = (\sin x)^3$$

$$y = \frac{\sin 5x}{2}$$

$$y = \sqrt{\sin x}$$

$$y = \sin x^2$$

$$y = \sin \sqrt{x}$$

$$y = \sin^2 x$$

$$y = 6 \sin x^3$$

1.-	$y = \sin 2x$	$y' = 2 \cos 2x$
2.-	$y = \sin \frac{x}{5}$	$y' = \frac{1}{5} \cos \frac{x}{5}$
3.-	$y = \frac{\sin 5x}{2}$	$y' = \frac{1}{2} \cdot 5 \cos 5x = \frac{5 \cos 5x}{2}$
4.-	$y = \sin x^2$	$y' = 2x \cos x^2$
5.-	$y = \sin^2 x$	$y' = 2 \sin x \cdot \cos x$
6.-	$y = \sin(3x^5)$	$y' = 15x^4 \cos(3x^5)$
7.-	$y = (\sin x)^3$	$y' = 3(\sin x)^2 \cos x$
8.-	$y = \sqrt{\sin x}$	$y' = \frac{1}{2\sqrt{\sin x}} \cdot \cos x = \frac{\cos x}{2\sqrt{\sin x}}$
9.-	$y = \sin \sqrt{x}$	$y' = \frac{1}{2\sqrt{x}} \cos \sqrt{x}$
10.-	$y = 6 \sin x^3$	$y' = 18x^2 \cos x^3$

4. Deriva les funcions

$$y = e^{5x}$$

$$y = 5e^{8x}$$

$$y = e^{\sin x}$$

$$y = x^2 e^{3x}$$

$$y = 6x^3 \cos x$$

$$y = 4e^{5x+1}$$

$$y = e^{34,7}$$

$$y = e^{2,8x}$$

$$y = \sin x \cdot \cos x$$

$$y = \sin 2x \cdot \cos 3x$$

Handwritten solutions for the first 10 problems:

- $y = e^{5x}$ → $y' = 5e^{5x}$
- $y = 5e^{8x}$ → $y' = 40e^{8x}$
- $y = e^{\sin x}$ → $y' = \cos x e^{\sin x}$
- $y = x^2 e^{3x}$ → $y' = 2x e^{3x} + 3x^2 e^{3x} = x e^{3x} (2 + 3x)$
- $y = 6x^3 \cos x$ → $y' = 18x^2 \cos x - 6x^3 \sin x$
- $y = 4e^{5x+1}$ → $y' = 20e^{5x+1}$
- $y = e^{34,7}$ → $y' = 0$
- $y = e^{2,8x}$ → $y' = 2,8 \cdot e^{2,8x}$
- $y = \sin x \cos x$ → $y' = \cos^2 x - \sin^2 x$
- $y = \sin 2x \cos 3x$ → $y' = 2 \cos 2x \cos 3x - 3 \sin 2x \sin 3x$

5. Deriva

$$y = x^3 \sin x$$

$$y = \frac{\sin x}{x}$$

$$y = x^2 - \sin x$$

$$y = x^3 e^x$$

$$y = e^{\sqrt{x}}$$

$$y = x \tan x$$

$$y = 7^x$$

$$y = 2^{6x}$$

$$y = 5^3$$

$$y = \sin^2 x \cdot \cos x$$

1. $y = x^3 \sin x$	$y' = 3x^2 \sin x + x^3 \cos x$
2. $y = \frac{\sin x}{x}$	$y' = \frac{x \cos x - \sin x}{x^2}$
3. $y = x^2 \cos x$	$y' = 2x \cos x - x^2 \sin x$
4. $y = x^3 e^x$	$y' = 3x^2 e^x + x^3 e^x = x^2 e^x (3+x)$
5. $y = e^{\sqrt{x}}$	$y' = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}$
6. $y = x \tan x$	$y' = \tan x + x(1 + \tan^2 x)$
7. $y = 7^x$	$y' = 7^x \ln 7$
8. $y = 2^{6x}$	$y' = 6 \cdot 2^{6x} \ln 2$
9. $y = 5^3$	$y' = 0$
10. $y = \sin^2 x \cos x$	$y' = 2 \sin x \cos^2 x - \sin^3 x$

6. Deriva

$$y = (x + \sin x)^3$$

$$y = \ln 7x$$

$$y = \ln x^2$$

$$y = \ln \sin x$$

$$y = \ln x \cdot \sin x$$

$$y = \ln(x^2 + x)$$

$$y = \ln^2 x$$

$$y = \ln \sqrt{x}$$

$$y = 7 \ln(7x+1)$$

$$y = \sin \ln x$$

1. $y = (x + \sin x)^3$	$y' = 3(x + \sin x)^2 (1 + \cos x)$
2. $y = \ln 7x$	$y' = \frac{1}{7x} \cdot 7 = \frac{1}{x}$
3. $y = \ln x^2$	$y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$
4. $y = \ln \sin x$	$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$
5. $y = \ln x \cdot \sin x$	$y' = \frac{1}{x} \sin x + \ln x \cos x$
6. $y = \ln(x^2 + x)$	$y' = \frac{1}{x^2 + x} (2x + 1) = \frac{2x + 1}{x^2 + x}$
7. $y = \ln^2 x$	$y' = 2 \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x}$
8. $y = \ln \sqrt{x}$	$y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$
9. $y = 7 \ln(7x+1)$	$y' = 7 \frac{1}{7x+1} \cdot 7 = \frac{49}{7x+1}$
10. $y = \sin \ln x$	$y' = \cos \ln x \cdot \frac{1}{x} = \frac{\cos \ln x}{x}$